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Date: Sat Dec 6, 2003 6:55:51 PM US/Pacific
To: Paul Zielinski <pzielins@ix.netcom.com>, David Gladstone <d1494@copper.net>
Subject: W. Pauli on Energy of Gravity

bcc

Reply to sarfatti@well.com not to above mindspring dummy address for out mail server from Caffe Roma wifi. Bay Club is better, do not need to trick the system to allow sending e-mail.

Since I am using new e-mail math fonts which may not show in primitive e-mail programs I also attach a pdf of the same message.

Paul
Followup on our meeting at Caffe Trieste last night. Looking at Pauli's 1921 article bearing in mind its early date.

Note prerequisites:

11. On forming tensor densities by multiplying ordinary tensors by $(\det g_{uv})^{1/2} = \sqrt{g}$

19. On Gauss & Stokes theorems in curved space-time.

20. eq (150b) for covariant 4-divergence of a second rank tensor density.

21. "free vectors" for "linear affine" subset of the nonlinear Diff(4) local symmetry group of gravity breaking global translational symmetry.
The ordinary 4-divergence of the affine tensor densities vanish corresponding to a limited kind of local current conservation for the linear subset of the full Diff(4) group, e.g. eq. (153) for a second rank affine tensor density.

One can then get a quasi-invariant "charge" and space integral affine "4-vector" in this linearized limit e.g. (156) for Ju.

But then the miracle happens:

Jv retains "this vector character also if we go from K to an arbitrary coordinate system K' which coincides with K outside the world canal .... the Jv are quite independent of the choice of coordinate system inside the world canal. It is interesting that starting with an affine tensor $U_v{}^u$ which behaves covariantly only under linear coordinate transformations, one should obtain by integration a set of quantities Jv which behave as a vector under a much more general transformation group. The vector Jv is distinguished from ordinary vectors by not being related to a given point. We shall follow Klein in calling it a 'free vector' [non-localized vector]"

Note (156) is the space integral of the tensor density (I am using math fonts that may not appear in Windows e-mail programs properly. It's a new Adobe standard now coming into play - GO MAC!

$J_v = \int U_v{}^4 \sqrt{g}\, d^3x$

So far this is close to Ch 20 of MTW.

Then go to 61 "The energy of the gravitational field"

Take the matter (source for bending) tensor density $T_{uv}(\text{Matter}) \sqrt{g}$

The Diff(4) covariant 4 divergence of the "wood" matter stress-energy density tensor density can be written from (150b) as

$\text{Div}^u[T_{uv}(\text{Matter})\sqrt{g}] = \partial[T_{uv}(\text{Matter})\sqrt{g}]/\partial x^u + [T_{rs}(\text{Matter})\sqrt{g}]\,\Gamma_v{}^{r}{}^{s}$

This vanishes in non-exotic vacuum when the Bianchi identities hold for the geometry "marble".

See (341a) for a slight variation on what I just wrote.

Then back to 57 eqs. (405) & (406)

to get a "flat" conservation equation with ordinary partial derivatives and with a local affine linear vacuum stress-energy density tensor density of the pure marble gravity field. One has to go back to 23 - we are back to the affine tensor densities
for the linear subset of Diff(4).

That is (406) is

$$\partial [T_{uv}(\text{Matter})\sqrt{g}]/\partial x^u + \partial [t_{uv}(\text{Geometry})\sqrt{g}]/\partial x^u = 0$$

With NONLOCAL "free vector" conserved

$$J_v = \int [T_{uv}(\text{Matter})\sqrt{g} + t_{uv}(\text{Geometry})\sqrt{g}]d^3x = \text{constant} \quad (447)$$

for a closed system where this tuv(Geometry) is a linear affine quasi-tensor not a full Diff(4) tensor.

How did Pauli explain this physically?

Einstein "proved that ... (447) for the total energy and momentum of a closed system are to a large extent, independent of the coordinate system, although the localization of the energy will in general be completely different for different coordinate systems ... ONE CANNOT ASSIGN ANY PHYSICAL MEANING TO THE tuv(Geometry)√g THEMSELVES, i.e. IT IS IMPOSSIBLE TO CARRY OUT A LOCALIZATION OF ENERGY AND MOMENTUM IN A GENERALLY COVARIANT AND PHYSICALLY SATISFACTORY WAY. BUT THE INTEGRAL EXPRESSIONS HAVE A DEFINITE PHYSICAL MEANING." p. 177 Dover paperback "Theory of Relativity" W. Pauli CAPS not in original. This is 1921 and obviously Pauli back then agrees with MTW 1973!

Pauli then discusses ASYMPTOTICALLY FLAT SPACE-TIME as the natural arena for the above in context of far field gravity waves. He also can do a similar thing for spatially closed universes in 62.

Note that Pauli mentions my rediscovered idea that

$$t_{uv}(\text{Geometry}) = (\text{String Tension})G_{uv}(\text{Einstein}) \quad \text{on p. 177}$$

so that, as a Diff(4) local tensor equation

$$T_{uv}(\text{Matter}) + t_{uv}(\text{Geometry}) = 0 \text{ LOCALLY}$$

in non-exotic vacuum.

This way of looking at Einstein's local field equations was apparently
first suggested by Lorentz and Levi-Civita as early as 1917!
Einstein did not like it because "the gravitational energy of a closed system
would always be zero, and that the maintenance of this value of the energy
does not require the continued existence of the system in some form or other."

However, I do not find Einstein's objection here at all convincing - for
reasons of metric engineering. Einstein's theory is the degenerate
limit of a bigger theory depending on Planck's h that allows metric
engineering. Einstein's theory only uses the parameters G and c.
Since he has $h = 0$ (no quantum theory) in 1918 he did not
realize that the stability of matter is an effect of h. What Einstein
says above is correct only in the limit $h = 0$. He could not know
this in 1918 nor could the 21 year old Pauli have known it in
1921 when the above was all written. It would be another 4 years
before Heisenberg et-al.

Also remember Paul I mentioned last night that the fundamental
clash here is that total energy-momentum are only conserved
for a globally flat translational group symmetry (Noether's theorem)
and that Einstein's gravity as curvature is the breaking of the
global translational symmetry i.e. locally gauging the translational

group with the metric huv = guv - Minkowskiuv as the compensating
local gauge tensor field, i.e. strain tensor of the world crystal distortion fields
d^u(x) = Lp^2 ∂(Goldstone Coherent Phase)/∂x^u