Non-equilibrium Physics for Dummies

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Start with the Boltzmann probability factor with partition function Z

$$p\left(E_N{}^j, N\right) = \frac{e^{-\left(E_N{}^j - \mu N\right)/k_B T}}{Z}$$
$$Z \equiv \sum_{E_N, N, j} p\left(E_N{}^j, N\right) \tag{1.1}$$

where j labels the energy eigenvalues for a complex system in thermodynamic equilibrium with whatever Lagrange multiplier constraints are appropriate to the problem. I shall show explicitly only the chemical potential Lagrange multiplier constraint[i] μ corresponding to conservation of the total number N of "normal fluid" particles in a superfluid that is above the critical phase transition temperature where the chemical potential vanishes because of the formation of the ground state[ii] condensate (BEC) that is a reservoir of particles.

$$\mu\left(T > T_c\right) \neq 0$$
$$\mu\left(T \leq T_c\right) = 0 \tag{1.2}$$

The appearance of the ground state condensate is a spontaneous broken symmetry of some symmetry group of the dynamical action for the complex many-particle system that P.W. Anderson calls "More is different." It is the uncertainty in the total number N of normal fluid particles that creates the macro-quantum long-range holographic θ phase coherence or "phase rigidity" because of the micro-quantum Heisenberg uncertainty relation between canonically conjugate dynamical variables that in this case is

$$\Delta N \Delta \theta \sim \frac{1}{2} \tag{1.3}$$

Consider an ideal Bose-Einstein gas with N identical particles of integer spin in thermal equilibrium. The mean fractional occupation number at total energy E_N is, suppressing j

$$\frac{\left\langle n\left(E_N\right)\right\rangle}{N} = \frac{1}{e^{\left(E_N - \mu N\right)/k_B T} - 1} \tag{1.4}$$

Take the limit $T \to 0$ that is automatically below the critical temperature where $\mu = 0$. Therefore in this limiting case

$$
\begin{aligned}
&\frac{\langle n(E_N)\rangle}{N} = \frac{1}{e^{E/k_B T} - 1} \to \delta_{Dirac}(E) \\
&\delta_{Dirac}(E) = 0, E \neq 0 \\
&\delta_{Dirac}(E) \to \infty, E = 0 \\
&\int_{-\infty}^{+\infty} \delta_{Dirac}(E) = 1
\end{aligned}
\tag{1.5}
$$

When we switch on inter-particle forces the Dirac delta function singular distribution is replaced by a smooth peaked function with only a finite fraction N_0 of the particles in the $E_N = 0$ state. In superfluid helium He^4 $N_0/N \approx 0.1$ as $T \to 0$ measured by inelastic neutron scattering.

This macro-quantum condensation is also described by the partial ODLRO[iii] partial factorization of low-order nonlocally entangled entropic reduced micro-quantum density matrices with the appearance of local un-entangled *zero entropy* macro-quantum holographic complex-numbered order parameters $\Psi(x)$ that can be thought of as giant quantum wave fields in ordinary space-time rather than in higher-dimensional configuration space. This precipitous drop in thermodynamic entropy from the collapse of a volume of higher-dimensional phase space for the complex system above the phase-transition spontaneous breaking of dynamical symmetry to a smaller volume of phase space is the cause of new emergent order in which the whole is qualitatively different from the reductionist mechanical form-independent simple sum of its parts. The appearance of inner consciousness, for example, must be such a giant quantum wave in our material bodies because the mind is a large thing not a tiny thing like a single electron. However, all living systems are way out of thermodynamic equilibrium. How are we to describe them? Now I show you a simple way to do that using a generalization of the Boltzmann factor.

All living systems are open systems with a flow of energy and matter through it. Let the energy flow through the open system be denoted by the power P. Think of the open system as a leaky resonator like the Fabry-Perot partially reflecting plates between a pumped active medium as in a laser. The leaky resonator, if left to itself, has a quality factor Q for total internal energy U with a leak rate dU/dt (for $P = 0$) defined as

$$
Q \equiv -\frac{U}{dU/dt} \tag{1.6}
$$

A negative Q means an active medium generating net output energy rather than simply absorbing input energy. The extraction of zero point energy from a pumped exotic vacuum region of space-time requires that the vacuum have a negative finite Q. Obviously $Q \to \infty$ in the ordinary non-exotic or "classical vacuum" with zero Cosmological Constant because its leak rate vanishes.

The effective "Frohlich energy"[iv] of an open system not in thermodynamic equilibrium is

$$E^* \equiv E_N - QP \tag{1.7}$$

Indeed we now see that Q is simply a Lagrange multiplier canonically conjugate to the through-put power flux P describing a constraint on the open non-equilibrium system. We will also see that a negative Q is vital to the formation of long-range holographic robust phase rigidity, i.e. non-classical macro-quantum coherence in which the usual probability rules of orthodox quantum measurement theory with the non-unitary von Neumann projection postulate for otherwise unitary time-evolution with signal nonlocality "passion at a distance"[v] détente with nonlocal entanglement and environmental decoherence is maintained. "More is different" robust macro-quantum phase rigidity[vi] is a protective barrier against the kind of environmental decoherence that W. Zurek writes about. It also means that Hawking's new solution[vii] of the black hole information paradox is almost certainly wrong because he, along with Lenny Susskind, assumes the universal validity of orthodox micro-quantum measurement theory for a black hole.

The generalized non-equilibrium Boltzman factor is then

$$\begin{aligned} p(E,N)_{nonequilibrium} &= \frac{e^{-(E_N - QP - \mu N - \ldots)/k_B T}}{Z^*} \\ Z^* &\equiv \sum_{E_N, N \ldots} p(E,N)_{nonequilibrium} \end{aligned} \tag{1.8}$$

Where the … means other constraints, like rotating a superconductor in the alleged Podkletnov/Ning-Li effect, for example. Let us now consider an ideal Bose-Einstein gas now not in thermodynamic equilbrium

$$\frac{\langle n(E_N)\rangle_{nonequilibrium}}{N} = \frac{1}{e^{(E_N - QP - \mu N)/k_B T} - 1} \tag{1.9}$$

Note that we cannot get a stable macro-quantum condensate in the open non-equilibrium system unless we have negative Q in the limit $P \to \infty$. Of course the system will self-destruct when the power flux through it is too large. The effective temperature of the non-equilibrium open system is with its critical power flux $P_{critical}$ for the emergence of an active "élan vital" state, with all other Lagrange multipliers vanishing, is deduced from

$$E_N - QP \to 0 \tag{1.10}$$

Which makes the denominator in (1.9) vanish causing all of the N particles to go into the macro-quantum coherent condensate in this ideal gas toy model. Note that the exponential in the denominator of (1.9) may not be less than one.

In this case a pumped macro-quantum coherent BEC condensate emerges because

$$\frac{\langle n(E_N)\rangle_{nonequilibrium}}{N} = \frac{1}{e^{(E_N - QP)/k_B T} - 1} \to \infty \tag{1.11}$$

For zero chemical potential spontaneously breaking normal fluid total number conservation that generates macro-quantum canonically conjugate phase coherence in the local giant quantum wave

$$\begin{aligned}
&\frac{E_N{}^j - QP}{E_N{}^j} = \left(1 - \frac{QP}{E_N{}^j}\right) \approx \left(1 - \frac{QP}{E_N{}^0}\right) \le \left(1 - \frac{QP}{E_N{}^1}\right) \le \left(1 - \frac{QP}{E_N{}^2}\right) \le \dots \\
&E_N{}^0 \le E_N{}^1 \le \dots \\
&j = 0,1,2,\dots
\end{aligned} \tag{1.12}$$

$$T_{nonequilibrium} \approx \frac{T_{equilibrium}}{\left(1 - \frac{QP}{E_N{}^0}\right)} \tag{1.13}$$

Remember that the ground state energy $E_N{}^0$ of this complex N-particle open non-equilibrium system is the smallest energy, which implies that it gives the dominant contribution to (1.13) for a given positive Q and P. Furthermore, in general for a nonlinear open system

$$Q = Q(P) \tag{1.14}$$

If we think of a laser, the critical power flux $P_{critical}$ at the boundary between a passive non-lasing open system and an active lasing open system is $T_{nonequilibrium} \to +\infty$ so that

$$1 - \frac{Q(P_{critical})P_{critical}}{E_N{}^0} \approx 0 \tag{1.15}$$

Beyond this critical threshold (1.13) changes to

$$\begin{aligned} &T_{nonequilibrium} \approx -\frac{T_{equilibrium}}{\left(1-\frac{QP}{E_N{}^0}\right)} \\ &Q<0 \\ &T_{nonequilibrium} \rightarrow 0- \\ &P \rightarrow \infty \end{aligned} \tag{1.16}$$

That is, the effective non-equilibrium temperature is negative for an active medium and it goes to zero with increasing pump power flux P assuming that the power flux does not destroy the system. This is very similar to a system on N qubits between a ground and excited state, e.g. with N spin 1/2 magnetic moments in an external magnetic field where the entropy S increases to a max at infinite temperature and then decreases to zero in the negative temperature region going from – infinity to – 0. We clearly see how the source of emergent complexity is in the decrease of entropy induced by the power flux through the open system.

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[i] If the system is rotating like Newton's famous bucket, there will be a Lagrange multiplier ϖ constraint corresponding to conservation of the total rotational angular momentum $\vec{J}$.

[ii] "ground state" for a system of N real on-mass-shell-particles, "vacuum" for a system of off-mass-shell virtual particles. The total energy E and momentum p are constrained by an equation on-mass-shell, but are not so constrained off-mass-shell where one has a susceptibility response correlation function whose rigid Fourier transform $\tilde{\chi}\left(\omega,\vec{k}\right)$ has support over the entire $\omega,\vec{k}$ space.

[iii] Off-Diagonal-Long-Range-Order

[iv] Herbert Frohlich first made a model like this in the special case of a biological membrane pictured as a lattice of electric dipoles pumped by external electrical energy.

[v] Abner Shimony's term.

[vi] Sakharov's "metric elasticity" = $G/8\pi c^4$ for the emergence of Einstein's gravity from the partial cohering of random micro-quantum zero point vacuum fluctuations is a particular example of the general phenomenon I am here describing for the first time in the history of physics in its fullness.

[vii] To be announced at GR17 Dublin, July 2004 where I am also giving a paper on this idea of emergent gravity from partial vacuum coherence.